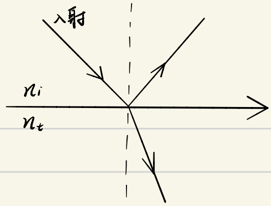
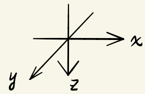




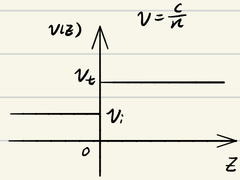
入射面为 x, z 平面 ($k_i^y = 0$)

波动方程

对偶场：$\vec{E}$ & $\vec{B}$

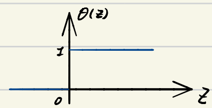
耦合关系：Maxwell 方程

解耦合：$\partial_t^2 \vec{E} = v^2(z)\, \nabla^2 \vec{E}$ + 边界条件



光源：入射光

波动方程：$\partial_t^2 \vec{E} = v^2(z) \nabla^2 \vec{E} + \vec{E}_i \cdot \theta(-z)$



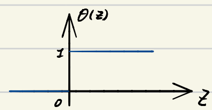
1) 当 $z<0$ 时 $\partial_t^2 \vec{E} = \frac{c^2}{n_i^2} \nabla^2 \vec{E} + \vec{E}_i$

2) 当 $z>0$ 时 $\partial_t^2 \vec{E} = \frac{c^2}{n_t^2} \nabla^2 \vec{E}$

3) 当 $z=0$ 时 $\begin{cases} \vec{E}_{x\text{-}y}(z=0^+) = \vec{E}_{x\text{-}y}(z=0^-) \\ \vec{B}_{x\text{-}y}(z=0^+) = \vec{B}_{x\text{-}y}(z=0^-) \end{cases}$ 电场切向连续 $\mu_i = \mu_t = 1$

波函数

预解式：$\vec{E} = (\vec{E}_i + \vec{E}_r) \cdot \theta(-z) + \vec{E}_t \cdot \theta(z)$



代入波动方程：验证解的形式，求解解的参数

$z<0$ 预解式成立，且 $\frac{\omega_r}{|k_r|} = \frac{c}{n_r}$

$z>0$ 预解式成立，且 $\frac{\omega_t}{|k_t|} = \frac{c}{n_t}$

$z=0$ $\begin{cases} \vec{E}_{x\text{-}y}(z=0^-) = \vec{E}_{x\text{-}y}(z=0^+) \Rightarrow \vec{E}_i^{x\text{-}y}(z=0) + \vec{E}_r^{x\text{-}y}(z=0) = \vec{E}_t^{x\text{-}y}(z=0) \\ \vec{B}_{x\text{-}y}(z=0^-) = \vec{B}_{x\text{-}y}(z=0^+) \Rightarrow \vec{B}_i^{x\text{-}y}(z=0) + \vec{B}_r^{x\text{-}y}(z=0) = \vec{B}_t^{x\text{-}y}(z=0) \end{cases}$

$\Downarrow$

$$\begin{cases} \omega_i = \omega_r = \omega_t & \text{同频、同色} \\ k_i^y = k_r^y = k_t^y = 0 & \text{共面} \\ k_i^x = k_r^x = k_r^x & \text{波矢匹配条件} \\ E_i^x + E_r^x = E_t^x, \quad E_i^y + E_r^y = E_t^y \\ B_i^x + B_r^x = B_t^x, \quad B_i^y + B_r^y = B_t^y \end{cases}$$

物理性质（方向、强度）

波的传播方向

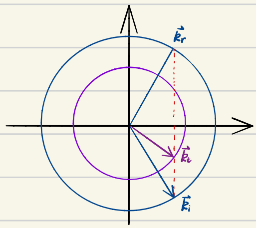
$$\omega_i=\omega_r=\omega_t \qquad k_i^y=k_r^y=k_t^y=0 \qquad k_i^x=k_r^x=k_t^x$$

Snell's Law

$$\begin{cases} \omega_i=\omega_r=\omega_t \Rightarrow \dfrac{|k_i|}{n_i}=\dfrac{|k_r|}{n_r}=\dfrac{|k_t|}{n_t} \\ k_i^x=k_r^x=k_t^x \Rightarrow |k_i|\sin\theta_i=|k_r|\sin\theta_r=|k_t|\sin\theta_t \end{cases}$$

$$\Rightarrow \begin{cases} n_i\sin\theta_i=n_r\sin\theta_r \Rightarrow \theta_i=\theta_r \quad (\text{折射}) \\ n_i\sin\theta_i=n_t\sin\theta_t \Rightarrow \text{Snell's Law} \quad (\text{反射}) \end{cases}$$

等波矢面法



$$|k_i|=|k_r|=\frac{\omega_i n_i}{c}$$

$$|k_t|=\frac{\omega_t n_t}{c}$$

全反射

等波面法表示

当 $|k_i|\sin\theta_i>|k_t| \Rightarrow \sin\theta_i>\dfrac{|k_t|}{|k_i|}=\dfrac{n_t}{n_i}$ 时发生全反射

解析表示

$$\vec{E}_t=\vec{E}_t^0\, e^{i(k_t^x x+k_t^y y-\omega_t t)}$$

$$\left.\begin{aligned} k_t^x&=k_i^x \\ (k_t^x)^2+(k_t^z)^2&=\frac{\omega_t^2 n_t^2}{c^2} \end{aligned}\right\} \Rightarrow k_t^z=\sqrt{\frac{\omega_t^2 n_t^2}{c^2}-(k_i^x)^2}$$

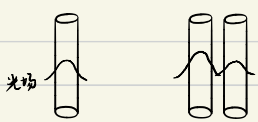
当 $k_i^x>|k_i|=\dfrac{\omega_t n_t}{c}$ 时，k_t^z 为虚数

波函数变为 $\vec{E}_t=\vec{E}_t^0\, e^{i(k_t^x x-\omega t)}\, e^{-\sqrt{(k_i^x)^2-\frac{\omega_t^2 n_t^2}{c^2}}\, z}$ 沿z方向e指数衰减

△ 全反射并非没有折射，而是折射光的强度沿z轴指数衰减

受阻的全反射

光纤　　　光纤1 光纤2



光纤1中的光向光纤2中泄漏

光的强度

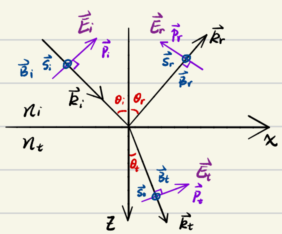
原则

Fresnell's Law 菲涅尔定律

利用电场和磁场的边界条件

电场 $\begin{cases} \vec{n}\times(\vec{E}_1-\vec{E}_2)=0 \\ \vec{n}\cdot(\vec{D}_1-\vec{D}_2)=\rho_f \end{cases}$ 磁场 $\begin{cases} \vec{n}\cdot(\vec{B}_1-\vec{B}_2)=0 \\ \vec{n}\times(\vec{H}_1-\vec{H}_2)=K_f \end{cases}$

特殊情形①



TM mode $\quad \vec{B}=(0,B_y,0) \quad \vec{E}=|E|\cdot\hat{p}$

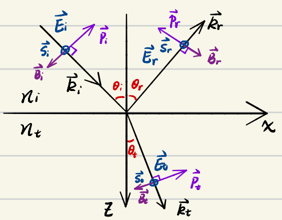
边界条件： $\begin{cases} E_i^x+E_r^x=E_t^x \Rightarrow |E_i|\cos\theta_i-|E_r|\cos\theta_r=|E_t|\cos\theta_t \\ B_i^y+B_r^y=B_t^y \Rightarrow |B_i|+|B_r|=|B_t| \end{cases}$

E-B关系： $|B_i|=\frac{|E_i|}{v_i}=\frac{|E_i|n_i}{c} \quad |B_r|=\frac{|E_r|n_i}{c} \quad |B_t|=\frac{|E_t|n_t}{c}$

Snell定律： $n_i\sin\theta_i=n_t\sin\theta_t \qquad \theta_i=\theta_t$

$\Rightarrow$ $$\frac{|E_r|}{|E_i|}=\frac{n_t\cos\theta_i-n_i\cos\theta_t}{n_t\cos\theta_i+n_i\cos\theta_t} \qquad \frac{|E_t|}{|E_i|}=\frac{2n_i\cos\theta_i}{n_t\cos\theta_i+n_i\cos\theta_t}$$

特殊情形②



TE mode $\quad \vec{E}=(0,E_y,0) \quad \vec{B}=|B|\cdot(-\hat{p})$

边界条件 $(\mu\approx1)$ $\begin{cases} E_i^y+E_r^y=E_t^y \Rightarrow |E_i|+|E_r|=|E_t| \\ B_i^x+B_r^x=B_t^x \Rightarrow -|B_i|\cos\theta_i+|B_r|\cos\theta_r=-|B_t|\cos\theta_t \end{cases}$

E-B关系： $|B_i|=\frac{|E_i|}{v_i}=\frac{|E_i|n_i}{c} \quad |B_r|=\frac{|E_r|n_i}{c} \quad |B_t|=\frac{|E_t|n_t}{c}$

Snell定律： $n_i\sin\theta_i=n_t\sin\theta_t \qquad \theta_i=\theta_t$

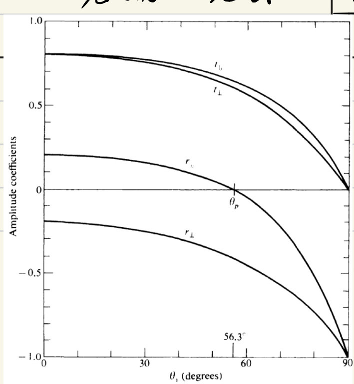
$\Rightarrow$ $$\frac{|E_r|}{|E_i|}=\frac{n_i\cos\theta_i-n_i\cos\theta_t}{n_i\cos\theta_i+n_i\cos\theta_t} \qquad \frac{|E_t|}{|E_i|}=\frac{2n_i\cos\theta_i}{n_i\cos\theta_i+n_t\cos\theta_t}$$

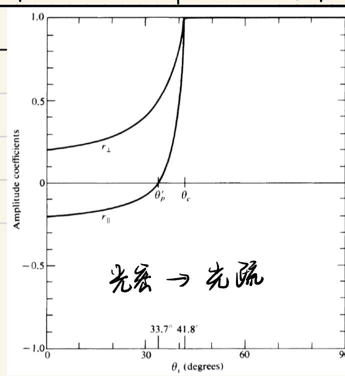
小结

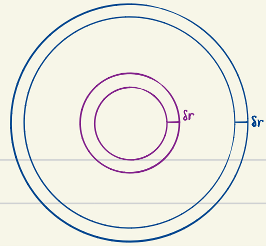
令 $m=\frac{\cos\theta_t}{\sin\theta_i} \qquad \rho=\frac{n_t}{n_i}$

	TM ($\hat{p}$分量)	TE ($\hat{S}$分量)
振幅反射率	$r_p=\frac{\rho-m}{\rho+m}$	$r_s=\frac{1-\rho m}{1+\rho m}$
振幅透射率	$t_p=\frac{2}{\rho+m}$	$t_s=\frac{2}{1+\rho m}$

光疏 → 光密

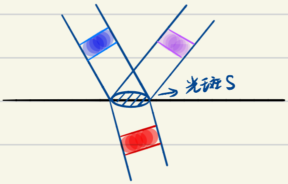






能量守恒	
物理背景	在t时刻空间某一体积内的光能传播到t+δt后，能量不变

物理系统



t时刻的体积

t+δt时刻在反射光

t+δt时刻在折射光

求解

① 求 V(t) 和 V(t+δt)

$$V_i(t) = S_i h_i = S h_i \cos\theta_i$$

$$V_r(t+\delta t) = S_r h_r = S h_r \cos\theta_r$$

$$V_t(t+\delta t) = S_t h_t = S h_t \cos\theta_t$$

$$\begin{cases} \dfrac{h_i}{v_i} = \dfrac{h_r}{v_r} = \dfrac{h_t}{v_t} \\ v = \dfrac{c}{n} \end{cases}$$

$$\Rightarrow n_i h_i = n_i h_r = n_t h_t$$

② 求电磁场能量密度

$$\rho_i = \frac{\varepsilon_i |E_i|^2}{2} + \frac{|B_i|^2}{2} = \varepsilon_i |E_i|^2 = \frac{n_i^2}{c^2}|E_i|^2$$

$$\rho_r = \varepsilon_i |E_r|^2 = \frac{n_i^2}{c^2}|E_r|^2 = \frac{n_i^2}{c^2} r^2 |E_i|^2$$

$$\rho_t = \varepsilon_t |E_t|^2 = \frac{n_t^2}{c^2}|E_t|^2 = \frac{n_t^2}{c^2} t^2 |E_i|^2$$

③ 验证 $U_i(t) = U_r(t+\delta t) + U_t(t+\delta t)$

$$U_i(t) = \rho_i V_i(t) = \frac{n_i^2}{c^2}|E_i|^2 \cdot S h_i \cos\theta_i = \frac{n_i h_i}{c^2} S |E_i|^2 \cdot n_i \cos\theta_i = I_i \cos\theta_i$$

$$U_r(t+\delta t) = \rho_r V_r(t+\delta t) = \frac{n_i^2}{c^2} r^2 |E_i|^2 \cdot S h_r \cos\theta_r = \frac{n_i h_i}{c^2} S |E_i|^2 \cdot r^2 n_i \cos\theta_i = I_r \cos\theta_i$$

$$U_t(t+\delta t) = \rho_t V_t(t+\delta t) = \frac{n_t^2}{c^2} t^2 |E_i|^2 \cdot S h_t \cos\theta_t = \frac{n_i h_i}{c^2} S |E_i|^2 \cdot t^2 n_t \cos\theta_t = I_t \cos\theta_t$$

$$I_i \cos\theta_i = I_r \cos\theta_r + I_t \cos\theta_t$$

强度反射率 $R = \frac{I_r}{I_i} = r^2$

强度透射率 $T = \frac{I_t}{I_i} = \frac{n_t}{n_i} t^2$

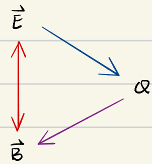
表 6－1 各种反射率和透射率的定义

	p 分量	s 分量
振幅反射率	$\tilde{r}_p = \tilde{E}'_{1p}/\tilde{E}_{1p}$ (6.13)	$\tilde{r}_s = \tilde{E}'_{1s}/\tilde{E}_{1s}$ (6.14)
强度反射率	$R_p = \frac{I'_{1p}}{I_{1p}} = \|\tilde{r}_p\|^2$ (6.15)	$R_s = \frac{I'_{1s}}{I_{1s}} = \|\tilde{r}_s\|^2$ (6.16)
能流反射率	$\mathscr{R}_p = \frac{W'_{1p}}{W_{1p}} = R_p$ (6.17)	$\mathscr{R}_s = \frac{W'_{1s}}{W_{1s}} = R_s$ (6.18)
振幅透射率	$\tilde{t}_p = \tilde{E}_{2p}/\tilde{E}_{1p}$ (6.19)	$\tilde{t}_s = \tilde{E}_{2s}/\tilde{E}_{1s}$ (6.20)
强度透射率	$T_p = \frac{I_{2p}}{I_{1p}} = \frac{n_2}{n_1}\|\tilde{t}_p\|^2$ (6.21)	$T_s = \frac{I_{2s}}{I_{1s}} = \frac{n_2}{n_1}\|\tilde{t}_s\|^2$ (6.22)
能流透射率	$\mathscr{T}_p = \frac{W_{2p}}{W_{1p}} = \frac{\cos i_2}{\cos i_1} T_p$ (6.23)	$\mathscr{T}_s = \frac{W_{2s}}{W_{1s}} = \frac{\cos i_2}{\cos i_1} T_s$ (6.24)

金属的折射和反射

波动方程

对偶场



M3 $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

欧姆定律 $\vec{J} = \sigma\vec{E}$

M4 $\nabla\times\vec{B} = \mu(\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t})$

解耦合

$$\left.\begin{array}{l}\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}\\ \nabla\times\vec{B} = \mu(\vec{J}+\varepsilon\frac{\partial\vec{E}}{\partial t})\end{array}\right\} \Rightarrow \left.\begin{array}{l}\varepsilon\mu\partial_t^2\vec{E} = \nabla^2\vec{E} - \mu\partial_t\vec{J}\\ \vec{J} = \sigma\vec{E}\end{array}\right\}$$

$$\Rightarrow \varepsilon\mu\partial_t^2\vec{E} = \nabla^2\vec{E} - \mu\sigma\partial_t\vec{E}$$ 耗散项

求解波函数

试探解 $E = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

代入波动方程 $-\omega^2\varepsilon\mu\vec{E} = -k^2\vec{E} + \mu\sigma(i\omega)\vec{E}$

$$\Rightarrow k^2 = \omega^2\left(\varepsilon\mu + \frac{i\mu\sigma}{\omega}\right)$$

$$\Rightarrow n^2 = \frac{c^2}{v^2} = c^2\frac{k^2}{\omega^2} = 1 + \frac{i\sigma}{\varepsilon\omega}$$

物理性质

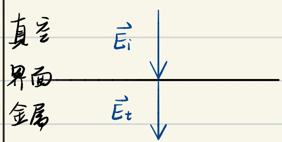
金属中的相速度

$$v_p = \frac{\omega}{|k|} = \sqrt{\varepsilon\mu + \frac{i\mu\sigma}{\omega}}$$

金属中的折射率

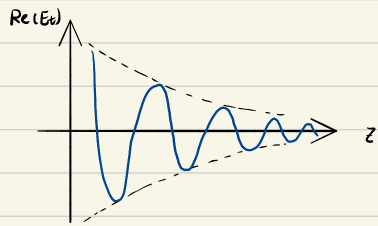
$$n = \sqrt{1 + \frac{i\sigma}{\varepsilon\omega}}$$

趋肤深度



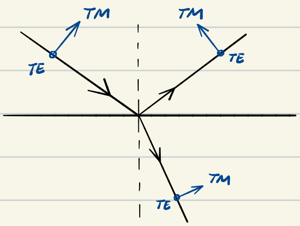
垂直入射时 $E_t = E_0 e^{ik_R z} e^{-k_I z}$ 复数

$I = I_0 e^{-2k_I z}$



$$\delta = \frac{1}{2}k_I = \frac{1}{\sqrt{2\omega\mu_0\sigma}}$$

折射&反射回顾



琼斯矢量 & 矩阵的描述

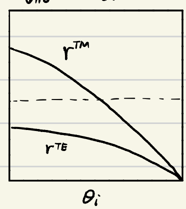
- 入、反、折射光的 J-矢量 $\begin{bmatrix}E_i^{TM}\\E_i^{TE}\end{bmatrix}_{入} \oplus \begin{bmatrix}E_r^{TM}\\E_r^{TE}\end{bmatrix}_{反} \oplus \begin{bmatrix}E_t^{TM}\\E_t^{TE}\end{bmatrix}_{透}$
- 折射&反射：J-矩阵

$$\begin{bmatrix}E_r^{TM}\\E_r^{TE}\end{bmatrix} = \begin{bmatrix}r_{TM} & 0\\0 & r_{TE}\end{bmatrix}\begin{bmatrix}E_i^{TM}\\E_i^{TE}\end{bmatrix} \qquad \begin{bmatrix}E_t^{TM}\\E_t^{TE}\end{bmatrix} = \begin{bmatrix}t_{TM} & 0\\0 & t_{TE}\end{bmatrix}\begin{bmatrix}E_i^{TM}\\E_i^{TE}\end{bmatrix}$$

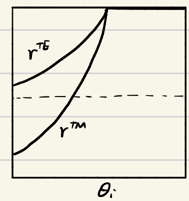
S-定律 & F-公式

- F-公式

疏 ⇒ 密



密 ⇒ 疏



- S-定律&F-公式适用所有平面波情形

折射&反射 ⇒ 偏振

概括

①利用B角入射产生/检测偏振光 ⇒ 偏振光

②利用全反射调节偏振光 ⇒ 波片

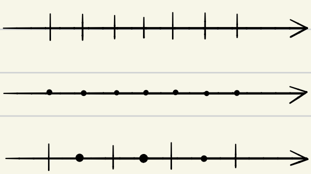
利用B-角(布鲁斯特角)做偏振片

B-角入射的回顾

定性描述

- B-角入射 ⇒ 反射光是纯TE光；透射光是TM+TE光
- 琼斯矢量 J-m $\begin{bmatrix}E_i^{TM}\\E_i^{TE}\end{bmatrix}_{入} \Rightarrow \begin{bmatrix}0\\rE_i^{TE}\end{bmatrix}_{反} + \begin{bmatrix}t_{TM}E_i^{TM}\\t_{TE}E_i^{TE}\end{bmatrix}_{透}$
- $\overleftrightarrow{J}_{反射} = \begin{bmatrix}0 & 0\\0 & r\end{bmatrix} \qquad \overleftrightarrow{J}_{透射} = \begin{bmatrix}t_{TM} & 0\\0 & t_{TE}\end{bmatrix}$

画图约定



TM

TE

TM + TE

矢量描述 (计算 i_B)

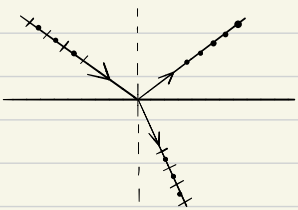
- 出发点：Snell定律 & Fresnel公式
- 关系①　$r_{TM}=r_{\parallel}=\frac{p-m}{p+m}=0 \Rightarrow p=m$

  $p=\frac{n_t}{n_i}$　　$m=\frac{\cos\theta_t}{\cos\theta_i}$

  $\Rightarrow \frac{\cos\theta_t}{\cos\theta_i}=\frac{n_t}{n_i}=\frac{\sin\theta_i}{\sin\theta_t} \Rightarrow \sin\theta_t\cos\theta_t=\sin\theta_i\sin\theta_t$

  $\Rightarrow \sin 2\theta_t=\sin 2\theta_i \Rightarrow \theta_t=\frac{\pi}{2}-\theta_i=\frac{\pi}{2}-i_B$

  B-角入射时，入射角和折射角互余



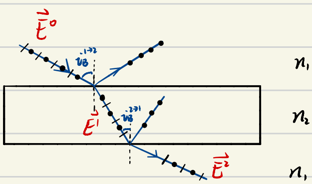
- 关系②　$\sin^2 i_B=\frac{n_t^2}{n_i^2+n_t^2} \Rightarrow i_B^{2\to1}+i_B^{1\to2}=\frac{\pi}{2}$

单层界面

$\overleftrightarrow{J}_{反}=\begin{bmatrix}0&0\\0&r_{TE}\end{bmatrix}$　　$\overleftrightarrow{J}_{透}=\begin{bmatrix}t_{TM}&0\\0&t_{TE}\end{bmatrix}$

偏振片

双层界面



描述 { 从 $n_1\to n_2$ 以 $i_B^{1\to2}$ 入射时，在第二个界面入射角恰等于 $i_B^{2\to1}$
经过两次折射，透射光中TE模式进一步减少

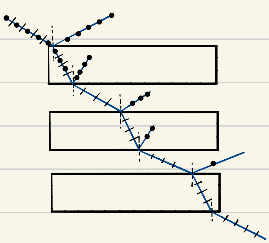
$\vec{E}^2=\overleftrightarrow{J}_{透}^{(2)}\vec{E}'=\overleftrightarrow{J}_{透}^{(2)}\overleftrightarrow{J}_{透}^{(1)}\vec{E}^0$　　证明

$\Rightarrow \overleftrightarrow{J}_{透}^{(1)+(2)}=\overleftrightarrow{J}_{透}^{(2)}\cdot\overleftrightarrow{J}_{透}^{(1)}$　(类似ABCD矩阵)

$\overleftrightarrow{J}_{透}^{(1)+(2)}=\begin{bmatrix}t_{\parallel}^{2\to1}&0\\0&t_{\perp}^{2\to1}\end{bmatrix}\begin{bmatrix}t_{\parallel}^{1\to2}&0\\0&t_{\perp}^{1\to2}\end{bmatrix}=\begin{bmatrix}t_{\parallel}^{2\to1}t_{\parallel}^{1\to2}&0\\0&\underbrace{t_{\perp}^{2\to1}t_{\perp}^{1\to2}}\end{bmatrix}$ 小于单层膜

$t_{\perp}^{2\to1}\cdot t_{\perp}^{1\to2} \ll t_{\perp}^{1\to2}$　经过两层膜之后，透射光中的TE模式↓↓

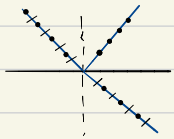
多层膜



经过多层膜B-角反射&折射

反射光纯TE　　透射光纯TM

$J_{反}^{多层}=\begin{bmatrix}0&0\\0&1\end{bmatrix}$　　$J_{透}^{多层}=\begin{bmatrix}1&0\\0&0\end{bmatrix}$

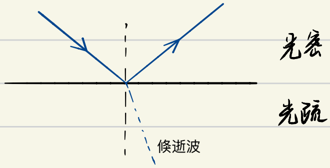


小结

$$J_{反} = \begin{bmatrix} 0 & 0 \\ 0 & r_{\perp} \end{bmatrix} \qquad J_{透} = \begin{bmatrix} t_{\parallel} & 0 \\ 0 & t_{\perp} \end{bmatrix}$$

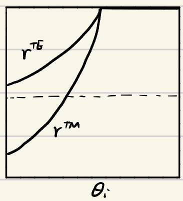
入射光	单层		多层	
	t	r	t	r
线偏	线	线(TE)	线(TM)	线(TE)
圆偏	椭			
椭偏	椭			
自然光	部分偏振			

利用全反射做波片



全反射回顾

密 ⇒ 疏



全反射时 $|r_{\parallel}| = |r_{\perp}| = 1$ $\quad r_{\parallel} = e^{i\phi_{\parallel}}$ $\quad r_{\perp} = e^{i\phi_{\perp}}$

证明：S-定律 ⇒ $n_t \sin\theta_t = n_i \sin\theta_i$ $\quad \sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

发生全反射时 $\sin\theta_t = \dfrac{n_i \sin\theta_i}{n_t} > 1$ （θ_t 是复数）

F-公式 ⇒ $r_{\parallel} = \dfrac{p-m}{p+m}$ 可证 $r_{\parallel} r_{\parallel}^{*} = 1$ $\quad |r_{\parallel}| = 1$

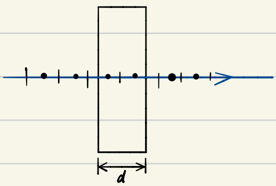
同理 $|r_{\perp}| = 1$ 且 $r_{\parallel}$ 和 $r_{\perp}$ 为复数

$r_{\parallel} = e^{i\phi_{\parallel}}$ $\qquad r_{\perp} = e^{i\phi_{\perp}}$

偏振调节

- $$\overleftrightarrow{J_r} = \begin{bmatrix} r_{\parallel} & 0 \\ 0 & r_{\perp} \end{bmatrix} \xrightarrow{全反射} \begin{bmatrix} e^{i\phi_{\parallel}} & 0 \\ 0 & e^{i\phi_{\perp}} \end{bmatrix} = e^{i\phi_{\parallel}} \begin{bmatrix} 1 & 0 \\ 0 & e^{i(\phi_{\perp}-\phi_{\parallel})} \end{bmatrix}$$
  $$= e^{i\phi_{\parallel}} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta\phi} \end{bmatrix}$$
- TM和TE模式的光在全反射时，有不同的相位效果（波片）
- $$\begin{bmatrix} E_r^{TM} \\ E_r^{TE} \end{bmatrix} = \overleftrightarrow{J_r} \begin{bmatrix} E_i^{TM} \\ E_i^{TE} \end{bmatrix} = e^{i\phi_{\parallel}} \begin{bmatrix} E_i^{TM} \\ e^{i\Delta\phi} E_i^{TE} \end{bmatrix}$$

$$J_{波} = e^{i\phi}\begin{bmatrix}1 & 0\\ 0 & e^{i\Delta\phi}\end{bmatrix}$$



波片

- 功能：对两个互相垂直的偏振方向，诱导不同的相位延迟
- 特征指标：快轴

  对于TM、TE模式的光，相位积累(光程)少的轴是快轴

  考虑一个波片，波片中 $v_{TE} > v_{TM} \Rightarrow k_{TE} < k_{TM}$ $\left(v = \frac{\omega}{k}\right)$

  光程 $\phi_{TE} = k_{TE} d < k_{TM} d = \phi_{TM}$

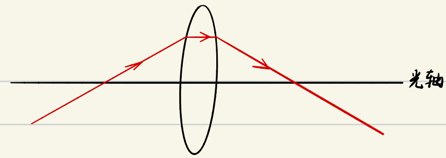
  e.g. 若快轴为TE轴，则 $\phi_{TM} > \phi_{TE}$ $\Delta\phi = \phi_{TE} - \phi_{TM} < 0$

  $$\overleftrightarrow{J} = e^{i\phi}\begin{bmatrix}1 & 0\\ 0 & e^{i(\phi_{TE}-\phi_{TM})}\end{bmatrix} = e^{i\phi}\begin{bmatrix}1 & 0\\ 0 & e^{i\Delta\phi}\end{bmatrix}$$

- example

入射光	$\frac{1}{8}$波片 ~ $\Delta\phi = \frac{2\pi}{8} = \frac{\pi}{4}$	$\frac{1}{4}$波片 ~ $\Delta\phi = \frac{2\pi}{4} = \frac{\pi}{2}$
线偏 $\begin{bmatrix}1\\2\end{bmatrix}$	椭偏 $\begin{bmatrix}1\\2e^{i\frac{\pi}{4}}\end{bmatrix}$ (一般)	椭偏 $\begin{bmatrix}1\\2e^{i\frac{\pi}{2}}\end{bmatrix}$ (一般)
圆偏 $\begin{bmatrix}1\\i\end{bmatrix}$	椭偏 $\begin{bmatrix}1\\e^{i\frac{3\pi}{4}}\end{bmatrix}$	线偏 $\begin{bmatrix}1\\e^{\pm i\pi}\end{bmatrix} \sim \begin{bmatrix}1\\-1\end{bmatrix}$
椭偏	椭偏	椭偏
自然	自然	自然

几何光学



物理系统

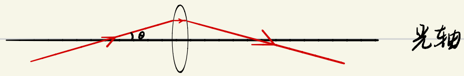
光

光沿直线传播

Snell's Law $\begin{cases} \theta_i = \theta_r \\ n_i \sin\theta_i = n_t \sin\theta_t \end{cases}$

光学元器件

存在一个对称轴 $\Rightarrow$ 光轴



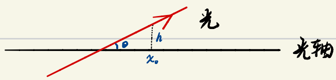
傍轴近似

$\theta \approx 0^\circ$

数学建模

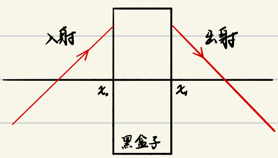
光的建模

用最简单的一串数字完整地描述光线



$\Rightarrow (h, \theta)_{x_0} \Rightarrow \begin{pmatrix} h \\ \theta \end{pmatrix}_{x_0}$

光学元器件



原则

光学元器件的作用是把一束入射光变成一束出射光

$\Leftrightarrow$ 建立一个从入射光 $\begin{pmatrix} h_{\text{入}} \\ \theta_{\text{入}} \end{pmatrix}_{x_0}$ 到出射光 $\begin{pmatrix} h_{\text{出}} \\ \theta_{\text{出}} \end{pmatrix}_{x_1}$ 的函数

数学模型

$$\begin{cases} h_{\text{出}} = f_1(h_{\text{入}}, \theta_{\text{入}}) \\ \theta_{\text{出}} = f_2(h_{\text{入}}, \theta_{\text{入}}) \end{cases} \xRightarrow[f_1, f_2\text{是线性函数}]{\text{线性近似}} \begin{cases} h_{\text{出}} = a h_{\text{入}} + b \theta_{\text{入}} \\ \theta_{\text{出}} = c h_{\text{入}} + d \theta_{\text{入}} \end{cases}$$

ABCD矩阵

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

光学元器件由 (a, b, c, d) 4个数字确定

$$\begin{bmatrix} h_{\text{出}} \\ \theta_{\text{出}} \end{bmatrix}_{x_1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} h_{\text{入}} \\ \theta_{\text{入}} \end{bmatrix}_{x_0}$$

傍轴近似

线性近似 (f_1, f_2)

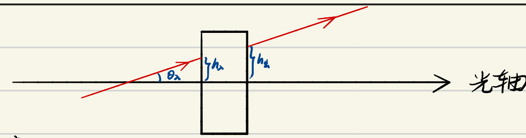
光学元器件举例

单个光学元器件

真空

$$\begin{bmatrix} 1 & x_1-x_0 \\ 0 & 1 \end{bmatrix}$$

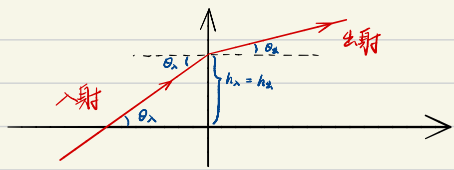
最简单的光学元器件



$$\begin{cases} h_出 = h_入 + (x_1-x_0)\tan\theta_入 \\ \theta_出 = \theta_入 \end{cases} \xRightarrow[\tan\theta_入 \sim \theta_入]{傍轴近似} \begin{cases} h_出 = h_入 + (x_1-x_0)\theta_入 \\ \theta_出 = 0\cdot h_入 + \theta_入 \end{cases}$$

折射界面

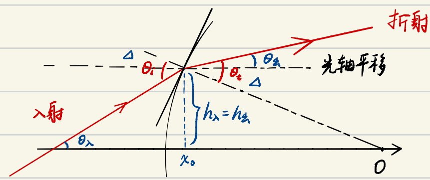
$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_入}{n_出} \end{bmatrix}$$



$$\begin{cases} h_出 = h_入 \\ n_入\sin\theta_入 = n_出\sin\theta_出 \end{cases} \xRightarrow{傍轴近似} \begin{cases} h_出 = h_入 + 0\cdot\theta_入 \\ \theta_出 = 0\cdot h_入 + \frac{n_入}{n_出}\theta_入 \end{cases}$$

球形界面

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_入}{n_出}-1\right) & \frac{n_入}{n_出} \end{bmatrix}$$

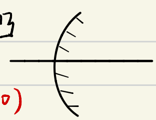


曲面曲率很小

$\Rightarrow R \gg$ 所有特征尺寸

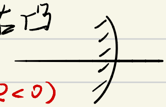
$$\begin{cases} h_出 = h_入 \\ \begin{cases} \theta_i = \theta_入 + \Delta \\ \theta_t = \theta_出 + \Delta \\ \sin\Delta = \frac{h_入}{R} \xrightarrow{曲率小} \Delta = \frac{h_入}{R} \\ n_入\sin\theta_i = n_出\sin\theta_t \xRightarrow{傍轴近似} n_入\theta_i = n_出\theta_t \end{cases} \end{cases} \Longrightarrow \begin{cases} h_出 = h_入 + 0\cdot\theta_入 \\ \theta_出 = \frac{1}{R}\left(\frac{n_入}{n_出}-1\right)h_入 + \frac{n_入}{n_出}\theta_入 \end{cases}$$

左凹 $(R>0)$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_i}{n_0}-1\right) & \frac{n_i}{n_0} \end{bmatrix}$$

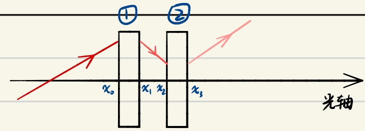
右凹 $(R<0)$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{|R|}\left(1-\frac{n_i}{n_0}\right) & \frac{n_i}{n_0} \end{bmatrix}$$

组合光学元器件(共轴)

抽象系统



$$\begin{bmatrix} a_{12} & b_{12} \\ c_{12} & d_{12} \end{bmatrix} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \otimes \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

推导

入 $\begin{bmatrix} h_{\text{入}} \\ \theta_{\text{入}} \end{bmatrix}$ 中 $\begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix}_{x_1} \to \begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix}_{x_2}$ 出 $\begin{bmatrix} h_{\text{出}} \\ \theta_{\text{出}} \end{bmatrix}$

$$\begin{bmatrix} h_{\text{出}} \\ \theta_{\text{出}} \end{bmatrix}_{x_3} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix}_{x_2}$$

$$\begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix}_{x_2} = \begin{bmatrix} 1 & x_2-x_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix}_{x_1}$$

$$\begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix}_{x_1} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} h_{\text{入}} \\ \theta_{\text{入}} \end{bmatrix}_{x_0}$$

$$\Rightarrow \begin{bmatrix} h_{\text{出}} \\ \theta_{\text{出}} \end{bmatrix}_{x_3} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} 1 & x_2-x_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} a_{12} & b_{12} \\ c_{12} & d_{12} \end{bmatrix} \begin{bmatrix} h_{\text{入}} \\ \theta_{\text{出}} \end{bmatrix}$$

光元①+真空+光元②

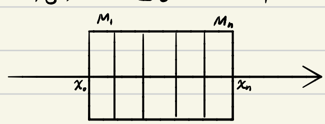
$$\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} 1 & x_2-x_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

光元①+光元②

$$\lim_{x_1 \to x_2} (\text{光元①}+\text{真空}+\text{光元②}) \Rightarrow \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

推广

任意共轴光学元器件的组合

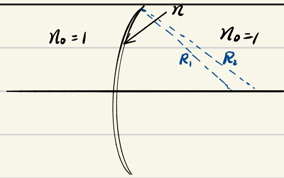


$$M_i \sim \begin{bmatrix} a_i & b_i \\ c_i & d_i \end{bmatrix}$$

整体 $\begin{bmatrix} a_{total} & b_{total} \\ c_{total} & d_{total} \end{bmatrix} = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix} \begin{bmatrix} a_{n-1} & b_{n-1} \\ c_{n-1} & d_{n-1} \end{bmatrix} \cdots \begin{bmatrix} a_1 & c_1 \\ b_1 & d_1 \end{bmatrix}$

举例

薄凹透镜

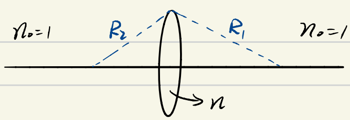


$$M_{透镜} = \begin{bmatrix}1&0\\ \frac{1}{R}\left(\frac{n_1}{n_2}-1\right) & \frac{n_1}{n_2}\end{bmatrix}_{R_2}\begin{bmatrix}1&0\\ \frac{1}{R}\left(\frac{n_1}{n_2}-1\right) & \frac{n_1}{n_2}\end{bmatrix}_{R_1} = \begin{bmatrix}1&0\\ \frac{1}{R_2}\left(\frac{n}{n_0}-1\right) & \frac{n}{n_0}\end{bmatrix}\begin{bmatrix}1&0\\ \frac{1}{R_1}\left(\frac{n_0}{n}-1\right) & \frac{n_0}{n}\end{bmatrix}$$

$$= \begin{bmatrix}1&0\\ \frac{1}{R_2}(n-1) & n\end{bmatrix}\begin{bmatrix}1&0\\ \frac{1}{R_1}\left(\frac{1}{n}-1\right) & \frac{1}{n}\end{bmatrix} = \begin{bmatrix}1&0\\ (n-1)\left(\frac{1}{R_2}-\frac{1}{R_1}\right) & 1\end{bmatrix} = \begin{bmatrix}1&0\\ -\frac{1}{f} & 1\end{bmatrix}$$

透镜焦距 f $\quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

薄凸透镜



$$M_{透镜} = \begin{bmatrix}1&0\\ \frac{1}{R_2}\left(1-\frac{n_1}{n_2}\right) & \frac{n_1}{n_2}\end{bmatrix}\begin{bmatrix}1&0\\ \frac{1}{R_1}\left(\frac{n_1}{n_2}-1\right) & \frac{n_1}{n_2}\end{bmatrix} = \begin{bmatrix}1&0\\ \frac{1}{R_2}\left(1-\frac{n}{n_0}\right) & \frac{n}{n_0}\end{bmatrix}\begin{bmatrix}1&0\\ \frac{1}{R_1}\left(\frac{n_0}{n}-1\right) & \frac{n_0}{n}\end{bmatrix}$$

$$= \begin{bmatrix}1&0\\ \frac{1}{R_2}(1-n) & n\end{bmatrix}\begin{bmatrix}1&0\\ \frac{1}{R_1}\left(\frac{1}{n}-1\right) & \frac{1}{n}\end{bmatrix} = \begin{bmatrix}1&0\\ (1-n)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & 1\end{bmatrix} = \begin{bmatrix}1&0\\ -\frac{1}{f} & 1\end{bmatrix}$$

焦距 f $\quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

偏振

波函数

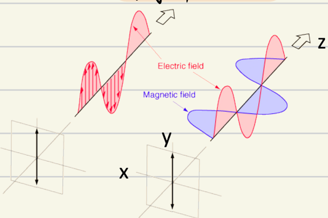
$$\partial_t^2 \vec{E} = \frac{1}{\mu\varepsilon}\nabla^2\vec{E} \iff \begin{cases} \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu\varepsilon\frac{\partial^2 E_x}{\partial t^2} \\ \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu\varepsilon\frac{\partial^2 E_y}{\partial t^2} \\ \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu\varepsilon\frac{\partial^2 E_z}{\partial t^2} \end{cases}$$

(E_x, E_y, E_z) 可以取不同的波函数

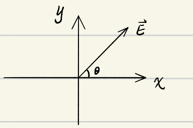
自然光（无偏振光）

各方向无优越性

线偏振光



$$\begin{cases} E_x = E\cos\theta = E_0\cos\theta\, e^{i(kz-wt)} = E_0^x e^{i(kz-wt)} \\ E_y = E\sin\theta = E_0\sin\theta\, e^{i(kz-wt)} = E_0^y e^{i(kz-wt)} \end{cases}$$

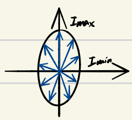


$$\begin{bmatrix} E_x(z,t) \\ E_y(z,t) \end{bmatrix} = \begin{bmatrix} E_0^x \\ E_0^y \end{bmatrix} e^{i(kz-wt)} = E_0 \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} e^{i(kz-wt)} = E_0^x \begin{bmatrix} 1 \\ \tan\theta \end{bmatrix} e^{i(kz-wt)}$$

琼斯矢量 $\sim \left\{ \begin{bmatrix} E_0^x \\ E_0^y \end{bmatrix}; \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}; \begin{bmatrix} 1 \\ \tan\theta \end{bmatrix} \right\}$

部分偏振光

介于自然光和线偏光之间

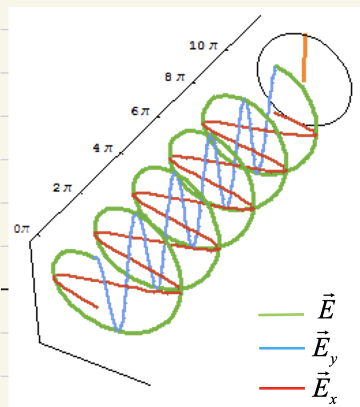


偏振度 $P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

$P = 0$ （自然光/非偏振光）

$P = 1$ （线偏振光）

圆偏振光

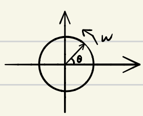


$$\begin{cases} E_x = E_0\cos(kz-wt) = Re[E_0 e^{i(kz-wt)}] \\ E_y = E_0\cos(kz-wt \pm \frac{\pi}{2}) = Re[E_0 e^{i(kz-wt\pm\frac{\pi}{2})}] \end{cases}$$

$$\begin{bmatrix} E_x(z,t) \\ E_y(z,t) \end{bmatrix} = E_0 \begin{bmatrix} 1 \\ e^{\pm i\frac{\pi}{2}} \end{bmatrix} e^{i(kz-wt)}$$

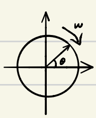
琼斯矢量

逆时针（左旋圆偏振光）



$$\begin{cases} E_x = E_0\cos(kz-wt) \\ E_y = E_0\sin(kz-wt) \end{cases}$$
$$= E_0\cos(kz-wt-\frac{\pi}{2})$$

顺时针（右旋圆偏振光）



$$\begin{cases} E_x = E_0\cos(kz-wt) \\ E_y = -E_0\sin(kz-wt) \end{cases}$$
$$= E_0\cos(kz-wt+\frac{\pi}{2})$$